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Kuis Pak Suhud

1). Diberikan $F = 2xy^2\hat{i} + xyz\hat{j} + yz^2\hat{k}$

Dapatkan : a) $\nabla \times F$ b). $\nabla \times (\nabla \times F)$ di $P(0,1,2)$

Jawab : a. $\nabla \times F = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(2xy^2\hat{i} + xyz\hat{j} + yz^2\hat{k}\right)$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy^2 & & yz^2 \end{vmatrix}$$

$= (z^2 - xy)\hat{i} + (0 - 0)\hat{j} + (yz - 4xy)\hat{k}$

$= z^2\hat{i} - xy\hat{i} + yz\hat{k} - 4xy\hat{k}$

$= 4\hat{i} - 0\hat{i} + 0\hat{j} + 2\hat{k} - 0\hat{k}$

Substitusi $P(0,1,2) \longrightarrow \nabla \times F\Big|_{(0,1,2)} = 4\hat{i} + 0\hat{j} + 2\hat{k}$

Jawab : $\nabla \times F\Big|_{P(0,1,2)} = 4\hat{i} + 2\hat{k}$

b). $\nabla \times (\nabla \times F)$

$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left((z^2 - xy)\hat{i} + 0\hat{j} + (yz - 4xy)\hat{k}\right)$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 - xy & 0 & yz - 4xy \end{vmatrix}$$

$= (z - 4x)\hat{i} + (-4y - 2z)\hat{j} + (-x)\hat{k}$

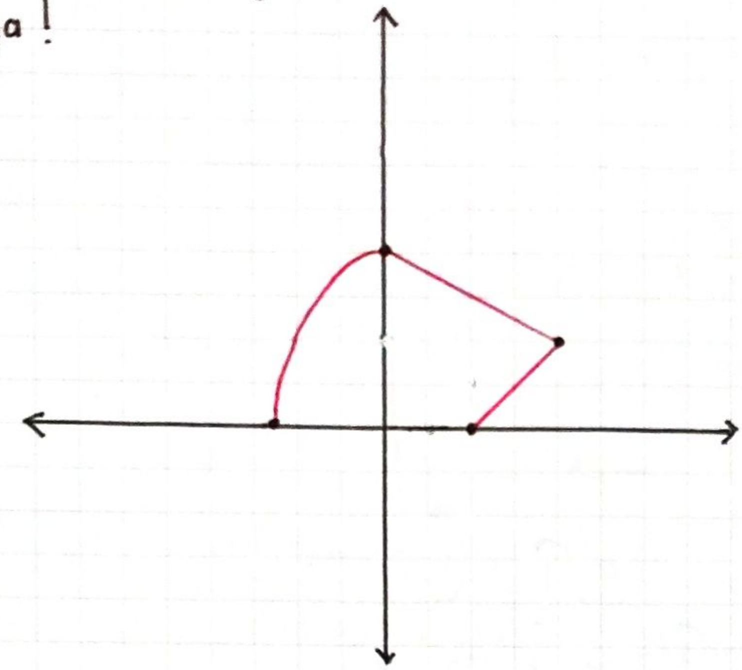
$= z\hat{i} - 4x\hat{i} - 4y\hat{j} - 2z\hat{j} - x\hat{k}$

Substitusi $p(0,1,2) \rightarrow = (2)\hat{i} - (-8)\hat{j}$

$= 2\hat{i} + 8\hat{j}$

Jawab = $\nabla \times (\nabla \times F)\Big|_{P(0,1,2)} = 2\hat{i} + 8\hat{j}$

2). Hitung $\int_C F \cdot dr$ di mana $F = 3x^2y\,\hat{i} - y\sqrt{x}\,\hat{j}$ dan C adalah lintasan melalui garis lurus dari $(1,0)$ ke $(2,1)$, kemudian garis lurus dari $(2,1)$ ke $(0,2)$, dan kemudian parabola $y = 2-x^2$ dari $(0,2)$ ke $(-\sqrt{2},0)$. Gambarkan juga lintasan C-nya!



▶ Menentukan Persamaan Garis

- Garis I : $(1,0)$ ke $(2,1)$ $\rightarrow$

$$\frac{y_2-y_1}{y-y_1} = \frac{x_2-x_1}{x-x_1}$$

$$\frac{1}{y} = \frac{1}{x-1}$$

$$x-1 = y$$

- Garis II : $(2,1)$ ke $(0,2)$ $\rightarrow$

$$\frac{y_2-y_1}{y-y_1} = \frac{x_2-x_1}{x-x_1}$$

$$\frac{1}{y-1} = \frac{-2}{x-2}$$

$$x-2 = 2-2y$$

$$y = \frac{x-4}{-2}$$

▶ Menghitung Integral Garis

- Garis $y = x-1$

$dy = dx$

$$\int_C F \cdot dr = \int_C (3x^2y\,\hat{i} - y\sqrt{x}\,\hat{j}) \cdot (dx\,\hat{i} + dy\,\hat{j})$$

$$= \int_C 3x^2(x-1)\,dx - \int_C (x-1)\sqrt{x}\,dx$$

$$= \int_1^2 3x^3 - 3x^2\,dx - \int_1^2 x\sqrt{x} - \sqrt{x}\,dx$$

$$= \frac{3}{4}x^4 - x^3 \Big|_1^2 - \frac{2}{5}x^2\sqrt{x} - \frac{2}{3}x\sqrt{x}\Big|_1^2$$

$$= \left[(12-8) - \left(\frac{3}{4}-1\right)\right] - \left[\left(\frac{8}{5}\sqrt{2} - \frac{4}{3}\sqrt{2}\right) - \left(\frac{2}{5} - \frac{2}{3}\right)\right]$$

$$= \left[4 + \frac{1}{4}\right] - \left[\frac{4\sqrt{2}}{15} + \frac{4}{15}\right]$$

$$= \frac{17}{4} - \left(\frac{4+4\sqrt{2}}{15}\right)$$

- Garis $y = \dfrac{x-4}{-2}$

$$dy = -\frac{1}{2}dx$$

$$\int_c F.\,dr = \int_c \left(3x^2y\,\hat{i} - y\sqrt{x}\,\hat{j}\right)\left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$= \int_c \left(3x^2\left(\frac{x-4}{-2}\right)\hat{i} - \frac{(x-4)\sqrt{x}}{-2}\hat{j}\right)\left(dx\,\hat{i} - \frac{1}{2}dx\,\hat{j}\right)$$

$$= \int_2^0 -\frac{3}{2}x^3 + 6x^2 - \frac{1}{4}x\sqrt{x} + \sqrt{x}\;dx$$

$$= -\frac{3}{8}x^4 + 2x^3 - \frac{1}{10}x^2\sqrt{x} + \frac{2}{3}x\sqrt{x}\,\Big|_2^0$$

$$= -\left[-6 + 16 - \frac{2\sqrt{2}}{5} + \frac{4}{3}\sqrt{2}\right]$$

$$= -\left[10 + \frac{14}{15}\sqrt{2}\right]$$

$$= -10 - \frac{14}{15}\sqrt{2}$$

- Kurva $y = 2 - x^2 \longrightarrow dy = -2x\,dx$

$$\int_c F.\,dr = \int_c \left(3x^2y\,\hat{i} - y\sqrt{x}\,\hat{j}\right).\left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$= \int_c \left(3x^2y\,\hat{i} - y\sqrt{x}\,\hat{j}\right).\left(dx\,\hat{i} - 2x\,dx\,\hat{j}\right)$$

$$= \int_0^{-\sqrt{2}} \left[3x^2(2-x^2) - (2-x^2)\sqrt{x}\,(-2x)\right]dx$$

$$= \int_0^{-\sqrt{2}} 6x^2 - 3x^4 + 4x\sqrt{x} . -2x^3\sqrt{x}\;dx$$

$$= 2x^3 - \frac{3}{5}x^5 + 4.\frac{2}{5}x^2\sqrt{x} - \frac{4}{9}x^4\sqrt{x}\,\Big|_0^{-\sqrt{2}}$$

$$= -4\sqrt{2} + \frac{12}{5}\sqrt{2} + \frac{16}{5}\sqrt{(-\sqrt{2})} - \frac{16}{9}\sqrt{(-\sqrt{2})}$$

$$= \frac{-8}{5}\sqrt{2}. + \frac{64}{5}\sqrt{(-\sqrt{2})}$$

$$=$$

JOYKO

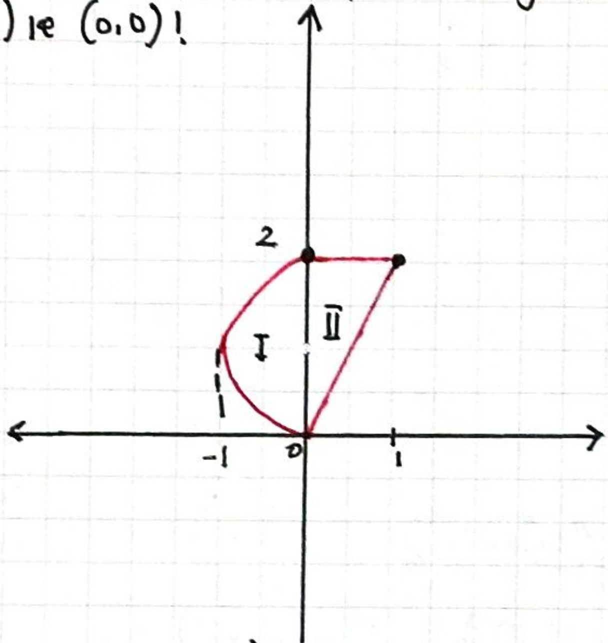
Nilai $\int_C F \cdot dr = \frac{17}{4} - \left(\frac{4+4\sqrt{2}}{15}\right) + \left(-10 - \frac{14}{15}\sqrt{2}\right) - \frac{8}{5}\sqrt{2} + \frac{64}{5}\sqrt{-\sqrt{2}}$

$$= \frac{17}{4} - \frac{4}{15} - 10 - \frac{4}{15}\sqrt{2} - \frac{14}{15}\sqrt{2} - \frac{24}{15}\sqrt{2} + \frac{64}{5}\sqrt{-\sqrt{2}}$$

$$= \frac{255 - 16 - 600}{60} - \frac{42}{15}\sqrt{2} + \frac{64}{5}\sqrt{-\sqrt{2}}$$

$$= \frac{-361}{60} - \frac{42}{15}\sqrt{2} + \frac{64}{5}\sqrt{-\sqrt{2}}$$

3). Dengan Teorema Green, hitung $\oint F \cdot dr$, jika $F = (x^2y - 3y^2)i - (x - y^3)j$ dan C adalah lintasan yang melalui garis lurus dari (0,0) ke (1,2), kemudian garis lurus dari (1,2) ke (0,2), dan selanjutnya melewati parabola $x = y^2 - 2y$ dari (0,2) ke (0,0)!



Diperoleh: $M = (x^2y - 3y^2)$ $N = (-x + y^3)$

$$\frac{dM}{dy} = x^2 - 6y \qquad \frac{dN}{dx} = -1$$

$$\oint F \cdot dr = \int_0^1 \int_0^2 -1 - (x^2 - 6y)\, dy\, dx$$

$$= \int_0^1 -y - x^2y + 3y^2 \Big|_0^2 dx$$

$$= \int_0^1 -2 - x^2 - 2 + 12\, dx$$

$$= \int_0^1 8 - x^2 dx$$

$$= 8x - \frac{1}{3}x^3 \Big|_0^1 = 8 - \frac{1}{3} = 7\frac{2}{3}$$